MAA American Mathematics Competitions

43rd Annual

AIME II

American Invitational Mathematics Examination II

Wednesday, February 12, 2025

INSTRUCTIONS

1. DO NOT TURN THE PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 15-question competition. All answers are integers ranging from 000 to 999, inclusive.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, protractors, and graph paper.
6. Figures are not necessarily drawn to scale.
7. You will have 3 hours to complete the competition once your competition manager tells you to begin.

The problems and solutions for this AIME were prepared by the
MAA AIME Editorial Board under the direction of:

Jonathan Kane and Sergey Levin, Co-Editors-in-Chief

The MAA AMC Office reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

A combination of your AIME score and your AMC 10/12 score is used to determine eligibility for participation in the USA (Junior) Mathematical Olympiad.

Problem 1:

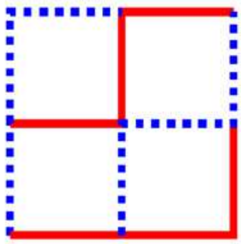
Six points A, B, C, D, E, and F lie in a straight line in that order. Suppose that G is a point not on the line and that $AC = 26$, $BD = 22$, $CE = 31$, $DF = 33$, $AF = 73$, $CG = 40$, and $DG = 30$. Find the area of $\triangle BGE$.

Problem 2:

Find the sum of all positive integers n such that $n + 2$ divides the product $3(n+3)(n^2+9)$.

Problem 3:

Four unit squares form a 2×2 grid. Each of the 12 unit line segments forming the sides of the squares is colored either red or blue in such a way that each unit square has 2 red sides and 2 blue sides. One example is shown below (red is solid, blue is dashed). Find the number of such colorings.



Problem 4:

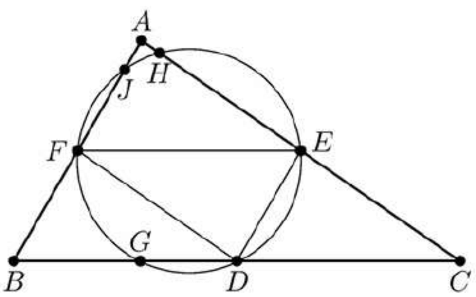
The product

$$\prod_{k=4}^{63} \frac{\log_k \left(5^{k^2-1}\right)}{\log_{k+1}\left(5^{k^2-4}\right)} = \frac{\log_4\left(5^{15}\right)}{\log_5\left(5^{12}\right)} \cdot \frac{\log_5\left(5^{24}\right)}{\log_6\left(5^{21}\right)} \cdot \frac{\log_6\left(5^{35}\right)}{\log_7\left(5^{32}\right)} \cdots \frac{\log_{63}\left(5^{3968}\right)}{\log_{64}\left(5^{3965}\right)}$$

is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

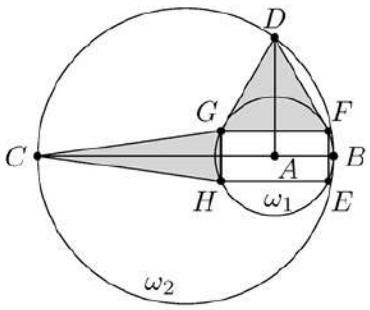
Problem 5:

Suppose $\triangle ABC$ has angles $\angle BAC = 84^\circ$, $\angle ABC = 60^\circ$, and $\angle ACB = 36^\circ$. Let D, E, and F be the midpoints of sides $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$, respectively. The circumcircle of $\triangle DEF$ intersects $\overline{BD}$, $\overline{AE}$, and $\overline{AF}$ at points G, H, and J, respectively. The points G, D, E, H, J, and F divide the circumcircle of $\triangle DEF$ into six minor arcs, as shown. Find $\widehat{DE} + 2\cdot\widehat{HJ} + 3\cdot\widehat{FG}$, where the arcs are measured in degrees.



Problem 6:

Circle ω_1 with radius 6 centered at point A is internally tangent at point B to circle ω_2 with radius 15. Points C and D lie on ω_2 such that $\overline{BC}$ is a diameter of ω_2 and $\overline{BC} \perp \overline{AD}$. The rectangle $EFGH$ is inscribed in ω_1 such that $\overline{EF} \perp \overline{BC}$, C is closer to $\overline{GH}$ than to $\overline{EF}$, and D is closer to $\overline{FG}$ than to $\overline{EH}$, as shown. Triangles $\triangle DGF$ and $\triangle CHG$ have equal areas. The area of rectangle $EFGH$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



Problem 7:

Let A be the set of positive integer divisors of 2025. Let B be a randomly selected subset of A. The probability that B is a nonempty set with the property that the least common multiple of its elements is 2025 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Problem 8:

From an unlimited supply of 1-cent coins, 10-cent coins, and 25-cent coins, Silas wants to find a collection of coins that has a total value of N cents, where N is a positive integer. He uses the so-called *greedy algorithm*, successively choosing the coin of greatest value that does not cause the value of his collection to exceed N. For example, to get 42 cents, Silas will choose a 25-cent coin, then a 10-cent coin, then 7 1-cent coins. However, this collection of 9 coins uses more coins than necessary to get a total of 42 cents; indeed, choosing 4 10-cent coins and 2 1-cent coins achieves the same total value with only 6 coins.

In general, the greedy algorithm *succeeds* for a given N if no other collection of 1-cent, 10-cent, and 25-cent coins gives a total value of N cents using strictly fewer coins than the collection given by the greedy algorithm. Find the number of values of N between 1 and 1000 inclusive for which the greedy algorithm succeeds.

Problem 9:

There are n values of x in the interval $0 < x < 2\pi$ where $f(x) = \sin\big(7\pi \cdot \sin(5x)\big) = 0$. For t of these n values of x, the graph of $y = f(x)$ is tangent to the x-axis. Find $n + t$.

Problem 10:

Sixteen chairs are arranged in a row. Eight people each select a chair in which to sit so that no person sits next to two other people. Let N be the number of subsets of the 16 chairs that could be selected. Find the remainder when N is divided by 1000.

Problem 11:

Let S be the set of vertices of a regular 24-gon. Find the number of ways to draw 12 segments of equal lengths so that each vertex in S is an endpoint of exactly one of the 12 segments.

Problem 12:
Let $A_1A_2A_3\ldots A_{11}$ be an 11-sided non-convex simple polygon with the following properties:

- For every integer $2 \le i \le 10$, the area of $\triangle A_iA_1A_{i+1}$ is equal to 1.
- For every integer $2 \le i \le 10$, $\cos(\angle A_iA_1A_{i+1}) = \frac{12}{13}$.
- The perimeter of the 11-gon $A_1A_2A_3\ldots A_{11}$ is equal to 20.

Then $A_1A_2 + A_1A_{11} = \frac{m\sqrt{n}-p}{q}$, where m, n, p, and q are positive integers, n is not divisible by the square of any prime, and no prime divides all of m, p, and q. Find $m+n+p+q$.

Problem 13:
Let $x_1, x_2, x_3, \ldots$ be a sequence of rational numbers defined by $x_1 = \frac{25}{11}$ and

$$x_{k+1} = \frac{1}{3}\left(x_k + \frac{1}{x_k} - 1\right)$$

for all $k \ge 1$. Then x_{2025} can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find the remainder when $m+n$ is divided by 1000.

Problem 14:
Let $\triangle ABC$ be a right triangle with $\angle A = 90^\circ$ and $BC = 38$. There exist points K and L inside the triangle such that

$$AK = AL = BK = CL = KL = 14.$$

The area of the quadrilateral $BKLC$ can be expressed as $n\sqrt{3}$ for some positive integer n. Find n.

Problem 15:
There are exactly three positive real numbers k such that the function

$$f(x) = \frac{(x-18)(x-72)(x-98)(x-k)}{x}$$

defined over the positive real numbers achieves its minimum value at exactly two positive real numbers x. Find the sum of these three values of k.